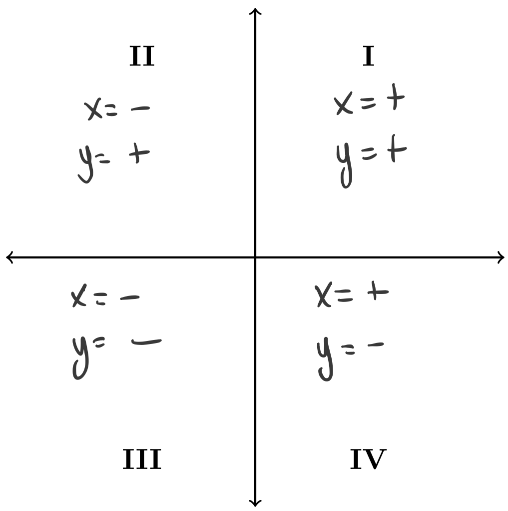
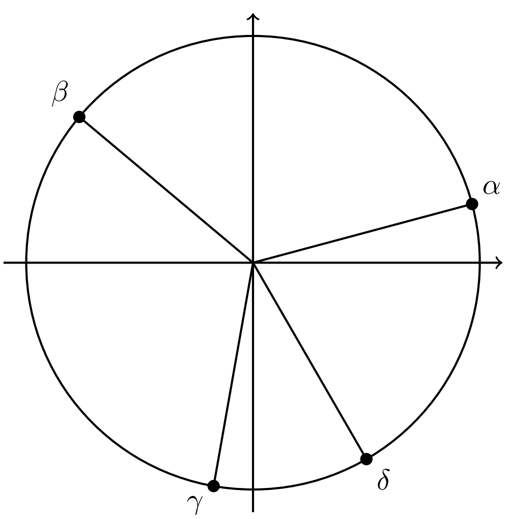
Chapter 5
Section 5.2-5.3

Main Topic # 1: [The Quadrants] First we will name the different parts of the coordinate plane



Learning Outcome # 1: [Know where sine and cosine are positive and negative]
Problem 1. Determine whether the sine and cosine of each angle shown below will be positive or negative.



(α) $\sin(\alpha) > 0$
$\cos(\alpha) > 0$

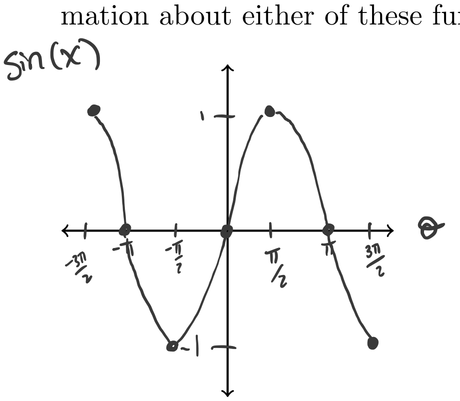
(β) $\sin(\beta) > 0$
$\cos(\beta) < 0$

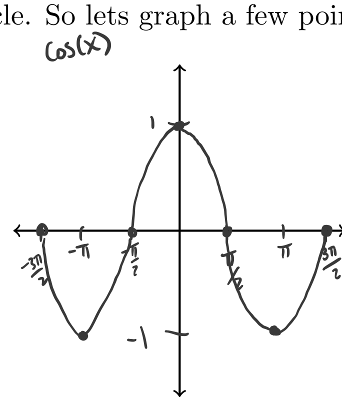
(γ) $\sin(\gamma) < 0$
$\cos(\gamma) < 0$

(δ) $\sin(\delta) < 0$
$\cos(\delta) > 0$

Main Topic # 2: [Sine and Cosine are Periodic functions]

The sine and cosine functions are very awesome functions. One awesome thing about them is that all the information about either of these functions is on the unit circle. So lets graph a few points.





\# what happens when we go past 2π or −2π

So we notice a few things:

(i) The **Domain** of both Sine and Cosine is all real numbers

(ii) The maximum value that either Sine or Cosine obtains is 1

(iii) The minimum value that either Sine or Cosine obtains is -1

(iv) The **Range** of both Sine and Cosine is the numbers between 1 and -1 that is: (-1,1)

some sort of reflection

(v) Both Sine and Cosine are both reflective over the line $y = 0$

(vi) Once go around the circle we start our values over that is:

$$\sin(\theta + 2\pi) = \sin(\theta)$$
$$\cos(\theta + 2\pi) = \cos(\theta)$$

Periodic Function

A function, $f(x)$ is **Periodic** if it repeats itself.
More technically this means there is a number, call it p so that

$$f(x + p) = f(x)$$

we call such a p **the period** of the function f.

So from the work above we see that both $\sin(x)$ and $\cos(x)$ are a periodic functions and they both have a period of 2π.

Amplitude of a periodic function

A periodic function, f which has a maximum, call it M and a minimum, call it m then we say the **Amplitude of f** is the finite number:

$$\frac{M - m}{2}$$

when this happens we also say that f has finite amplitude.

So from the work above we see that the amplitude of $\sin(x)$ and $\cos(x)$ is both 1.

The Midline of a function

When a function, f, is reflective across a line $y = a$ we call this line the **midline**
An ugly picture:

w/ half its period

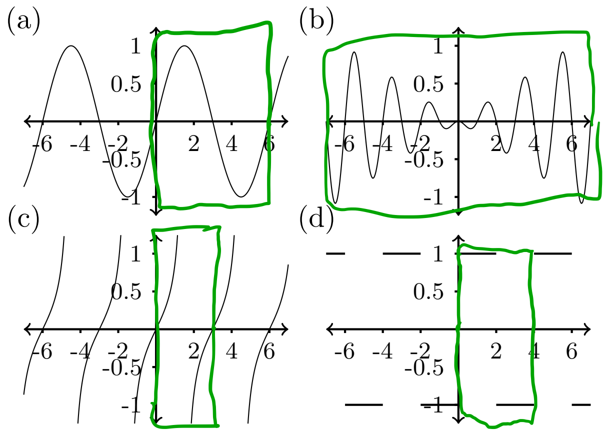
called its Frequency

told you it was ugly...

From the work above we see that both $\sin(x)$ and $\cos(x)$ have a midline of $y = 0$.

Learning Outcome # 2: [Identifying period, midline and amplitude]

Problem 2. Determine whether or not the following graphs could represent periodic functions. If so, identify the corresponding period, midline, and amplitude.



(a) p=6
Amp=1
y=0

(b) Maybe p=14 (i.e. -7 to 7)
we don't know it could repeat after that...

(c) p=3

(d) p=4

Problem 3. Determine whether or not the following data sets could be modeled by a periodic function. If so, identify the corresponding period. If not, explain why not.

(a)

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	1	-1	3	-3	1	-1	3	-3	1	-1

p=4

(b)

t	1	2	3	4	5	6	7	8
$g(t)$	1	-1	2	-2	3	-3	4	-4

p= Not enough data
but could be 8

Learning Outcome # 3: [Using the values from the unit circle to calculate sine and cosine]

Problem 4. Mark each of the following angles on the provided circle below. Then find the **exact** values of sine and cosine for each angle. [Hint: It might be helpful to use the unit circle.]

(a) $\pi/4$

$\sin(\pi/4) = \frac{\sqrt{2}}{2}$

$\cos(\pi/4) = \frac{\sqrt{2}}{2}$

(b) $2\pi/3$

$\sin(2\pi/3) = \frac{\sqrt{3}}{2}$

$\cos(2\pi/3) = -1/2$

(c) $-120°$

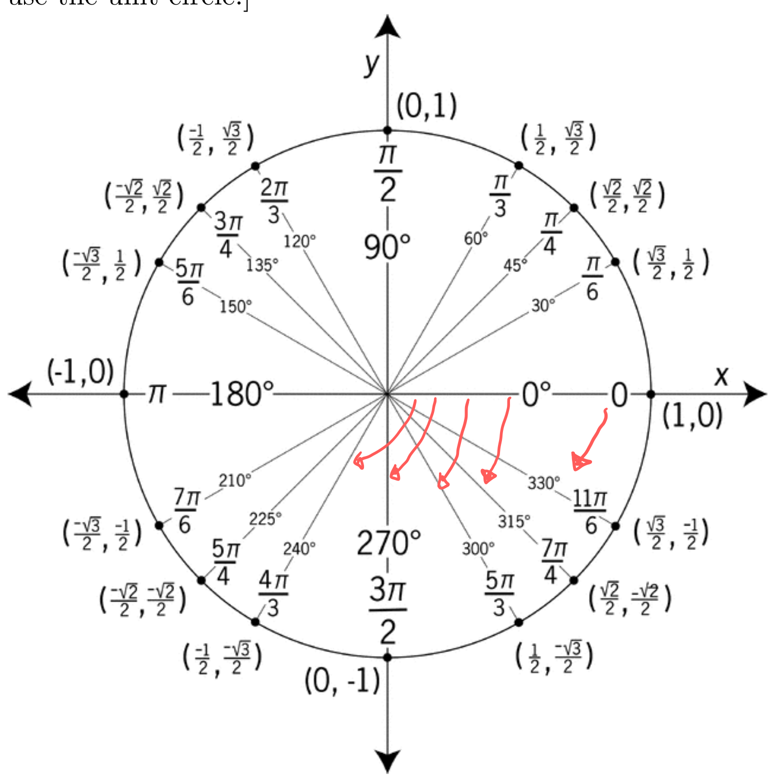
$\sin(-120°) = \frac{-\sqrt{3}}{2}$

$\cos(-120°) = -1/2$

(d) $270°$

$\sin(270°) = -1$

$\cos(270°) = 0$



Learning Outcome # 4: [Knowing how to shift sine and cosine functions]

Problem 5. Consider the generalized cosine function $h(x) = A\cos(B(x-h)) + k$

$= A\sin(B(x-h)) + k$

(a) What is the period of $h(x)$?

This is horizontal Stretch/compression remember its flipped:

$P = \frac{2\pi}{|B|}$

$P = \frac{2\pi}{B}$

$B = \frac{2\pi}{P}$

(b) What is the midline of $h(x)$?

this is a vertical shift

$y = k$

know these Transforms!

(c) What is the amplitude of $h(x)$?

this is a vertical Stretch/compression

$|A|$

Problem 6. Identify the period, midline, and amplitude of each of the trigonometric functions below. Include a sketch of each.

(a) $7\sin(t) - 8$

period $= 2\pi$

Amplitude $= |7| = 7$

Mid line $=$ $y = -8$

(b) $-2\sin\left(\frac{\pi}{2}t\right)$

period $= \frac{2\pi}{(\pi/2)} = \frac{2\pi}{1}\cdot\frac{2}{\pi} = \frac{4\pi}{\pi} = 4$

Amplitude $= |-2| = 2$

Mid line $=$ $y = 0$

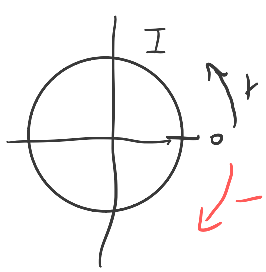
(c) $\cos(8\pi t) - 1$

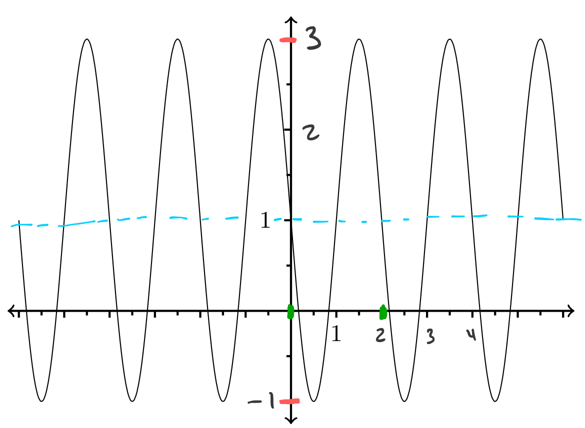
period $= \frac{2\pi}{8\pi} = \frac{1}{4}$

Amplitude $= 1$

Mid line $=$ $y = -1$

Problem 7. Consider the following graph.





Amplitude $= 2$

mid line $=$ $y = 1$

period $= 2$

$B = \frac{2\pi}{2} = \pi$

(a) Write a formula for the graph above using sine.

$-2\sin(\pi x) + 1$

(b) Write a formula for the graph above using cosine.

horizontal Shift $= \frac{\pi}{2}$ or $-\frac{\pi}{2}$

$-2\cos\left(\pi\left(x - \frac{\pi}{2}\right)\right) + 1$

Problem 8. In the function $A\sin(B(x-h)) + k$, the value h is called the *phase shift*. What is the phase shift of the function $\sin(Bx - \phi)$?

$\phi = B\cdot h$ i.e. $h = \frac{\phi}{B}$